Set theory - Winter semester 2016-17

Problems	Prof. Peter Koepke
Series 10	Dr. Philipp Schlicht

Problem 37 (6 points)**.** Suppose that κ is a cardinal with $\mathrm{cof}(\kappa) > \omega$.

(1) Suppose that μ, κ are regular cardinals and $f \colon \mu \to \kappa$ is a cofinal strictly increasing continuous function. Show that $f[C]$ is club in κ for every club C in μ.

(2) Suppose that κ is singular. Prove that for every regressive function $f \colon \kappa \to \kappa$, there is a stationary subset S of κ such that $\mathrm{ran}(f \restriction S)$ is bounded below κ.

(3) Suppose that κ is regular and A is a non-stationary subset of κ. Prove that there is a regressive function $f \colon A \to \kappa$ such that for all $\gamma < \kappa$, the set $\{\alpha \mid f(\alpha) \leq \gamma\}$ is bounded below κ.

Problem 38 (2 points)**.** Suppose that $(L, <)$ is a linear order and κ is a cardinal with $\mathrm{card}(\{x \in L \mid x < y\}) < \kappa$ for all $y \in L$. Show that $\mathrm{card}(L) \leq \kappa$.

Problem 39 (5 points)**.** Suppose that S is a stationary subset of a regular cardinal $\kappa > \omega$. A subset C of κ is called an S-*club* if it is unbounded in κ and $\sup(x) \in C$ for every $x \subseteq C$ with $\sup(x) \in S$. Prove that $\bigcap_{\alpha<\gamma} C_\alpha$ is an S-club for every sequence $\langle C_\alpha \mid \alpha < \gamma \rangle$ of S-clubs of length $\gamma < \kappa$.

Problem 40 (4 points)**.** Suppose that a train leaves at time 0 and is empty. It stops at every time α with $0 < \alpha < \omega_1$ and the following happens at every stop.

(1) First, one person leaves the train (we don't know which one), if the train is not empty. If the train is empty, nothing happens.

(2) Second, ω many people get on the train.

Prove that at time ω_1, the train is empty.

- Happy holidays! -

Due Friday, January 13, before the lecture.